SIMPLIFIED MODEL FOR EPICYCLIC GEAR INERTIAL CHARACTERISTICS

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ABSTRACT - For mechanical transmissions, all component gears have the same kinematics no matter the stage of the gear box. Consequently, the inertial characteristics of the mechanical transmission depend only on the square of the gear box ratio.
For an epicyclic gear box, the speeds of the components depend on the manner in which the power is transmitted through the planetary mechanisms; consequently, the inertial characteristics of the transmission depend on the structure of the epicyclic gear box.
The paper presents a simplified model for calculation of the inertia of an elementary epicyclic mechanism. The analysis includes the error estimation as well as the implementation of this simplified model into the analysis of the epicyclic gear box.

The usages of the planetary gear boxes become cover a larger area of drivelines for heavy vehicles, and for off-road vehicles too. Accompanied by a torque converter, the planetary gear box allows an automatic match of the engine output with the motion requirements which eases the driving and improves the running performances. The complexity of the kinematic structure of the transmissions including torque converter and planetary gear box derives from the existence of different moving elements for each stage as well as from the complex movement of the satellite gears belonging to the elementary epicyclic mechanisms.

For constant input speed, the kinematics of the planetary gear box may be studied using various methods as graph theory (6) or by extending "the traditional concept of a lever representation of a planetary gear set to one that includes negative lever ratios" (5). The methods were found suitable in a series of applications as in (1) and (4). A comprehensive approach is presented in (2) and (3) having as aim the calculation of speeds, torques, power flows and efficiency of the planetary gear box. All the paper cited above consider the stationary regime of the planetary mechanisms characterised by constant speed. Consequently, the inertia of the elements which form the planetary trains is neglected. This hypothesis proved to be too rough for solving some specific aspects such as the shifting process or the acceleration performances of the vehicle. In order to use the analysis method presented in (2) and (3) without neglecting the inertia of gears, a simplified model of epicyclic gear mechanism was developed, the main stages being presented below.

For the elementary epicyclic mechanism with 3DOF, the kinematics is fully described by the Willis relation:

$$\omega_1 + K \cdot \omega_2 - \left(1 + K\right) \cdot \omega_0 = 0 \quad (1)$$

where: ω - angular speed of the external elements noted with index 1 for sun gear, 2 for planet gear and 0 for carrier arm respectively; K - the constant of the epicyclic gear mechanisms:

$$K = \frac{z_2}{z_1},$$

where z represents the teeth number of the gear. The relation (1) give by integration and by derivation respectively:

$$\varphi_1 + K \cdot \varphi_2 - (1+K) \cdot \varphi_0 = 0 \quad (2)$$

$$\varepsilon_1 + K \cdot \varepsilon_2 - (1+K) \cdot \varepsilon_0 = 0 \quad (3)$$

For constant speed of the external elements:

$$\frac{\mathrm{d}\omega_1}{\mathrm{d}t} = 0; \quad \frac{\mathrm{d}\omega_2}{\mathrm{d}t} = 0; \quad \frac{\mathrm{d}\omega_0}{\mathrm{d}t} = 0,$$

the torque are given by the following relations:

$$\frac{M_1}{M_2} = \frac{1}{K}; \quad \frac{M_1}{M_0} = \frac{1}{-(1+K)}. \quad (4)$$

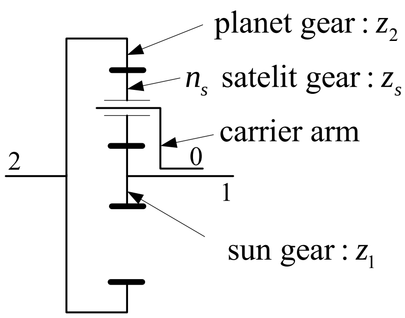

Figure 1 The structure of EI type epicyclic gear mechanism

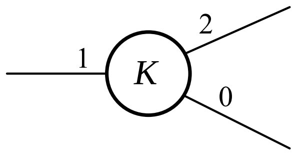

Figure 2 The nodal representation of the epicyclic gear mechanism

For the situation of acceleration, the existence of the inertia of gear produces a modified distribution of torques among the external elements. In order to specifically analyse the influence of inertia, the EI type gear mechanism is considered (see **Figure 1**).

The nodal representation is shown in **Figure 2 The nodal representation of the epicyclic gear mechanism** emphasising the external elements using the following notations: 1 for the sun gear, 2 for the planet gear and 0 for the carrier arm, respectively.

For this case, the Lagrange equation is applied considering the epicyclic gear mechanism as a system of mass points with two degree of freedom:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial}{\partial \omega_j} W\right) - \frac{\partial}{\partial \varphi_j} W = Q_j \quad j = 1, 2.$$

As generalized coordinates the angular displacement of elements 1 and 2 are adopted. Consequently, the total energy of the system is done by the following sum:

$$W = W_1 + W_2 + W_0 + W_s, \quad (5)$$

where the energy of the individual elements are:

$$W_1 = \frac{J_1 \cdot \omega_1^2}{2}; \quad W_2 = \frac{J_2 \cdot \omega_2^2}{2}; \quad W_0 = \frac{J_0^* \cdot \omega_0^2}{2}. \quad (6)$$

In the relation (6), the term J_0^* represents the inertia of the carrier arm itself, without satellites. The satellites have a complex movement consisting of a rotation around its axle with the speed ω_s and a rotation around the axle of sun gear with the speed ω_0:

$$\omega_s = \frac{2(\omega_1 - \omega_0)}{1-K}, \tag{7}$$

and:

$$\varphi_s = \frac{2(\varphi_1 - \varphi_0)}{1-K}; \quad \varepsilon_s = \frac{2(\varepsilon_1 - \varepsilon_0)}{1-K}. \tag{8}$$

For n_s satellites having the mass M_s and the moment of inertia J_s, the total energy of the satellites becomes:

$$W_s = n_s \cdot \left(\frac{J_s \cdot \omega_s^2}{2} + \frac{M_s \cdot R_0^2 \cdot \omega_0^2}{2} \right). \tag{9}$$

Introducing the relations (6) and (9) in the relation (5), it results the total energy of the epicyclic gear mechanism:

$$W = \frac{J_1 \cdot \omega_1^2}{2} + \frac{J_2 \cdot \omega_2^2}{2} + \frac{J_0^* \cdot \omega_0^2}{2} + n_s \cdot \left(\frac{J_s \cdot \omega_s^2}{2} + \frac{M_s \cdot R_0^2 \cdot \omega_0^2}{2} \right). \tag{10}$$

Taking into consideration the relation (7), the relation (10) becomes:

$$W = \frac{\omega_1^2}{2}\left[J_1 + \frac{4n_s J_s}{(1-K)^2} \right] + \frac{J_2 \omega_2^2}{2} + \frac{\omega_0^2}{2}\left[J_0^* + \frac{M_s R_0^2 \omega_0^2}{2} + \frac{4n_s J_s}{(1-K)^2} \right] - \left[\frac{4n_s J_s}{(1-K)} + \frac{J_2 (1+K)}{K^2} \right] \omega_0 \omega_1 . \tag{11}$$

The overall inertia of the carrier arm and n_s satellites is noted J_0:

$$J_0 = J_0^* + n_s \cdot M_s \cdot R_0^2 .$$

Using the above notation, the relation (11) becomes:

$$W = \frac{\omega_1^2}{2}\left[J_1 + \frac{4n_s \cdot J_s}{(1-K)^2} \right] + \frac{J_2 \cdot \omega_2^2}{2} + \frac{\omega_0^2}{2}\left[J_0 + \frac{4n_s \cdot J_s}{(1-K)^2} \right] - \frac{4n_s \cdot J_s \cdot \omega_0 \cdot \omega_1}{(1-K)} . \tag{12}$$

From the Willis relation results the speed of the planet gear:

$$\omega_2 = \frac{(1+K) \cdot \omega_0 - \omega_1}{K} .$$

Consequently, the relation (12) may be rewritten:

$$W = \frac{\omega_1^2}{2}\left[J_1 + \frac{4n_s \cdot J_s}{(1-K)^2} + \frac{J_2}{K^2} \right] + \frac{\omega_0^2}{2}\left[J_0 + \frac{4n_s \cdot J_s}{(1-K)^2} + \frac{J_2 (1+K)^2}{K^2} \right] - \left[\frac{4n_s \cdot J_s}{(1-K)} + \frac{J_2 (1+K)}{K^2} \right] \omega_0 \cdot \omega_1 \tag{13}$$

The derivation of the relation (13) gives:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial}{\partial \omega_1} W \right) = \varepsilon_1 \left[J_1 + \frac{4n_s \cdot J_s}{(1-K)^2} + \frac{J_2}{K^2} \right] - \varepsilon_0 \left[\frac{4n_s \cdot J_s}{(1-K)} + \frac{J_2 (1+K)}{K^2} \right] \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial}{\partial\omega_0}W\right)=-\varepsilon_1\left[\frac{4n_s\cdot J_s}{(1-K)}+\frac{J_2(1+K)}{K^2}\right]+\left[J_0+\frac{4n_s\cdot J_s}{(1-K)^2}+\frac{J_2(1+K)^2}{K^2}\right] \tag{15}$$

The elementary machine work is calculated using the relation:

$$\delta Q=M_1\cdot\delta\varphi_1+M_2\cdot\delta\varphi_2+M_0\cdot\delta\varphi_0\,, \tag{16}$$

and, taking into consideration the relation (2), it results:

$$\delta Q=\left(M_1-\frac{M_2}{K}\right)\cdot\delta\varphi_1+\left[M_0+\frac{(1+K)\cdot M_2}{K}\right]\cdot\delta\varphi_0\,. \tag{17}$$

Finally, the following relations result:

$$\varepsilon_1\left[J_1+\frac{4n_s\cdot J_s}{(1-K)^2}+\frac{J_2}{K^2}\right]-\varepsilon_0\left[\frac{4n_s\cdot J_s}{(1-K)}+\frac{J_2(1+K)}{K^2}\right]=M_1-\frac{M_2}{K} \tag{18}$$

$$-\varepsilon_1\left[\frac{4n_s\cdot J_s}{(1-K)}+\frac{J_2(1+K)}{K^2}\right]+\varepsilon_0\left[J_0+\frac{4n_s\cdot J_s}{(1-K)^2}+\frac{J_2(1+K)^2}{K^2}\right]=M_0+\frac{(1+K)\cdot M_2}{K} \tag{19}$$

The differential equation (18) and (19) describe accurately the working of the epicyclic gear mechanism taking into consideration the inertia of all elements which constitute the epicyclic gear.

Below is discussed the consequences of the hypothesis according to which the inertia of the satellites is neglected (but not their mass). This hypothesis is based on the observation that the inertia moment depends on the square of the gear radius, and the radii of the satellites are small compared with those of the sun gear or the planet gear.

Imposing the condition $J_S=0$, the equations (18) and (19) become:

$$\varepsilon_1\left[J_1+\frac{J_2}{K^2}\right]-\varepsilon_0\frac{J_2(1+K)}{K^2}=M_1-\frac{M_2}{K} \tag{20}$$

$$-\varepsilon_1\frac{J_2(1+K)}{K^2}+\varepsilon_0\left[J_0+\frac{J_2(1+K)^2}{K^2}\right]=M_0+\frac{(1+K)M_2}{K} \tag{21}$$

The equations (20) and (21) allow an approximate calculation of the reduced inertia moments. In order to evaluate the level of errors, the most significant situations are analysed below.

If the planet gear is blocked, it results:

$$\varepsilon_2=0;\quad \varepsilon_1=(1+K)\cdot\varepsilon_0 \tag{22}$$

Introducing the relations (22) in the relation (18), it results the exact distribution of torques acting on the external elements:

$$\varepsilon_0\left[(1+K)J_1+\frac{8n_s\cdot J_s}{(1-K)^2}\right]=M_1-\frac{M_2}{K}\,. \tag{23}$$

Using the relation (20) the approximate distribution results:

$$\varepsilon_0\left[(1+K)J_1\right]=M_1-\frac{M_2}{K}\,. \tag{24}$$

Supposing that the input element is the sun gear, the nodal representation of the epicyclic gear mechanism is presented in **Figure 3** where between the torques acting on the epicyclic gear mechanism the relation (4) applies:

$$\frac{M_1'}{M_2}=\frac{1}{K}.$$

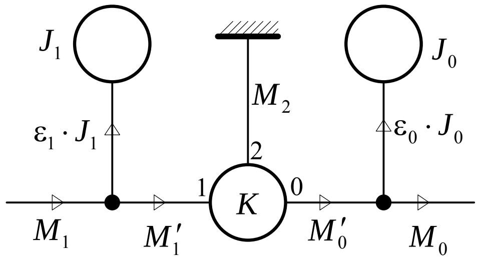

Figure 3 The nodal representation of the epicyclic gear mechanism

The balance of the torque on the ramified node gives, in general:

$$M_1' + J_1 \cdot \varepsilon_1 + M_1 = 0.$$

Supposing that $\varepsilon_1 > 0$ (the input shaft is accelerated), then:

$$\frac{M_1 - J_1\varepsilon_1}{M_2} = \frac{1}{K}. \tag{25}$$

Taking into consideration the relation (22), the relation (25) becomes:

$$\varepsilon_0\left[(1+K)J_1\right] = M_1 - \frac{M_2}{K} \tag{26}$$

which is identical with the relation (24).

The relative error generated by the hypothesis according to which the inertia of the satellites is neglected is done by the relation:

$$\Delta_2 = \frac{8n_s}{(1-K)^2}\frac{J_s}{J_1(1+K)}. \tag{27}$$

Similarly, if the sun gear is blocked, the relative error is done by the following relation:

$$\Delta_1 = \frac{4n_s}{(1-K)^2}\frac{J_s}{J_0 + \frac{J_2}{K}} \tag{28}$$

For the usually values of the parameters of the relations (27) and (28) ($n_s = 4...6$, $K = 2...4$) and considering the dimensions of the gears used in the planetary gear boxes, the relative errors are below 4%.

The analysis presented above allows the conclusions presented below.

The proposed model is based on the hypothesis according to which the inertia of the satellites is neglected; thus, the rotation of the satellites around their axes is neglected. Nevertheless, the mass of the satellites is taken into consideration being included into the relation of the equivalent inertia of the carrier arm:

$$J_0 = J_0^* + n_s M_s R_0^2.$$

The resulting relative error is considered acceptable.

The relations which describe the distribution of the torques acting on the external elements of the epicyclic gear mechanism are the following:

$$\frac{M_1 - J_1 \frac{\mathrm{d}\,\omega_1}{\mathrm{d}\,t}}{M_2 - J_2 \frac{\mathrm{d}\,\omega_2}{\mathrm{d}\,t}} = \frac{1}{K} \tag{29}$$

$$\frac{M_1 - J_1 \frac{\mathrm{d}\,\omega_1}{\mathrm{d}\,t}}{M_0 - J_0 \frac{\mathrm{d}\,\omega_0}{\mathrm{d}\,t}} = \frac{1}{-(1+K)} \tag{30}$$

The relations (29) and (30) allow the utilisation of the methods and algorithms presented in (2); thus, it becomes possible to use the same analysis methods for both stationary and transient regimes of the planetary gear boxes.

By the use of the simplified model for a given planetary gear box, the overall inertia of the gear box, reduced to the input shaft, may be determined for each stage using the nodal approach exposed in 10, without an unacceptable increase of the calculation volume.

REFERENCES

(1) Ashmore C., "New epicyclic gearbox series for 20To 40MW gas turbines", Gas Turbine World, 36(4), 30-31, 2006
(2) Ciobotaru T., Frunzeti D. Jäntschi L., "A Method for Analysing Epicyclic Gearboxes", International Journal of Automotive Technology, Vol. 11, No. 2, pp. 167−172, ISSN 1229-9138, 2010
(3) Ciobotaru T., Frunzeti D., Rus I., Jäntschi L., "Method for Analysing Multi-Path Power Flow Transmissions", Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, Vol. 224, Number 9/2010, p.1447-1454, ISSN 0954-4054 (Print), 2010
(4) Karaivanov A, Popov R., "Computer Aided Kinematic Analysis of Planetary Gear Trains of the 3k Type", Proceedings of the 3rd International Conference on Manufacturing Engineering (ICMEN), 1-3 October, Chalkidiki, Greece, 571-578, 2008
(5) Raghavan M., "Efficient Computational Techniques for Planetary Gear Train Analysis", 12th IFToMM World Congress, Besançon (France), June18-21, 1-5, 2007
(6) Tsai L. W., "Enumeration of Kinematic Structures According to Function", CRC Press LLC, ISBN 0-8493-0901-8, 155-182, 2001