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CINEMATIC ANALYSIS OF MULTICOUNTOUR MECHANISMS

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ABSTRACT - In this paper is presented a general cinematic analysis of multicountour mechanisms. These kind of mechanisms are found in the composure of systems that modifies the height of heat engine valves. These can also be composed of cam-follower mechanisms in which the cam and the follower have a plane-parallel movement. Beginning with the structural aspects and continuing by extending the cinematic analysis methods for plane bar mechanisms and cam mechanisms, one obtains a nonlinear equation system. The equations are based on the contour conditions and also on the tangency in contact point (of the cam-follower grooves) condition. Finally is obtained a nonlinearly equation system that is solvable through the Newton - Raphson method. Next are presented the determination relations for the velocities and accelerations of the cam-follower ensemble. In the end of the paper is presented the calculation algorithm proposed for the cinematic analysis of complex mechanisms with cams and bars.

INTRODUCTION

The mechanisms with many contours are usually plane mechanisms. They appear in the composition of the mechanisms with bars, but also in the composition of the complex mechanisms with cams. So the mechanism from fig. 1, is formed of: the plane quadrilateral mechanism *ABCD*, the tetrad *EFGHIJK* and the mechanism with the cam 2 and follower 6, both of them being in a plan parallel movement. In the composure of mechanism there are 8 mobile elements, 11 cinematic couples of fifth grade and one of fourth grade. It has four contours of mobility 1. Structurally, the contact couple from *M* can be replaced with a cinematic chain made of two fifth class couples (the equivalence theory of Gruebler and Harisberger). So, the mechanism from fig. 2 was obtained from the mechanism of fig. 1 by transforming the forth degree couple from *M* in the element 9 and the couples *M'* and *M''*. The mechanism still has four contours.

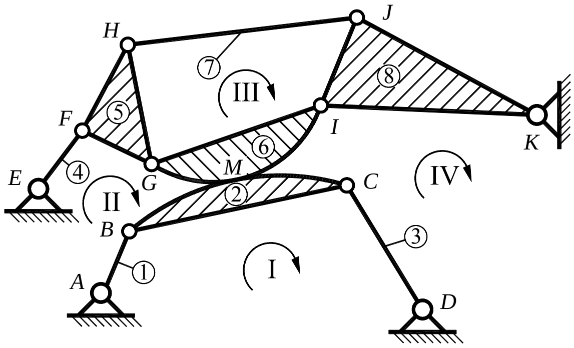

Fig. 1. Multicontour mechanism.

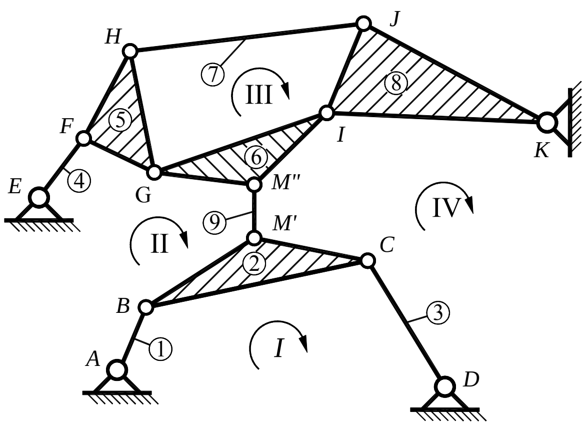

Fig. 2. Fundamental mechanism.

POSITIONAL ANALYSIS

In literature, the positional analysis of the mechanisms is done using few methods. Next we will present an analytic method based on the projecting contour method. The method is used on a four contours mechanism. For the mechanism from figure 3 we can identify the contours: *ABCD*, *ABMFE*, *EFGHI* and *EFGJK*.

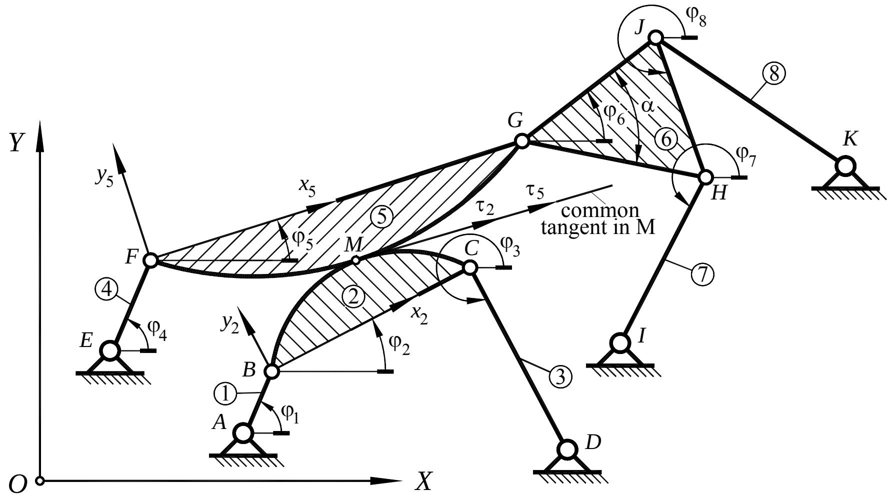

Fig. 3. Mechanism with four layers.

The mechanism is composed from the articulated quadrilateral ABCD that has a cam solidary with the rod 2. The Follower 5 is solidary with the rod of the mechanism with *EFGHIJK* triad. If there are no geometrical symmetries, the cam and the follower have a plane-parallel movement. The mechanism has one degree of mobility, the other elements having desmodrome movements.

In order to be able to write the contour equations, we will have to determine the coordinates of the point $M(X_M, Y_M)$ in the general reference system *XOY*, $M(x_2, y_2)$ in the local reference system x_2By_2 solidary with element 2 and $M(x_5, y_5)$ in the local reference system x_5Fy_5 solidary with element 5. We consider as known the profile of the cam and of the follower, the curbs being given by the parametric equations:

$$x_2 = x_2(\lambda);\; y_2 = y_2(\lambda) \qquad x_5 = x_5(\gamma);\; y_5 = y_5(\gamma) \tag{1}$$

Expressing the coordinates of the point *M* in all the three reference systems, we obtain the equalities:

$$\begin{aligned} X_M &= X_B + x_2\cos\varphi_2 - y_2\sin\varphi_2 = X_F + x_5\cos\varphi_5 - y_5\sin\varphi_5, \\ Y_M &= Y_B + x_2\sin\varphi_2 + y_2\cos\varphi_2 = Y_F + x_5\sin\varphi_5 + y_5\cos\varphi_5 \end{aligned} \tag{2}$$

The tangent in point *M* to the two curves (fig. 4) is defined by the vectors τ_2 and τ_5, with the components $\tau_2\left(\frac{\mathrm{d}x_2}{\mathrm{d}\lambda}, \frac{\mathrm{d}y_2}{\mathrm{d}\lambda}\right)$ and $\tau_5\left(\frac{\mathrm{d}x_5}{\mathrm{d}\gamma}, \frac{\mathrm{d}y_5}{\mathrm{d}\gamma}\right)$. The collinear condition is:

$$\tau_2 = \xi\tau_5 \tag{3}$$

that conducts to the relations

$$\begin{aligned} x_2'\cos\varphi_2 - y_2'\sin\varphi_2 &= \xi(x_5'\cos\varphi_5 - y_5'\sin\varphi_5) \\ x_2'\sin\varphi_2 + y_2'\cos\varphi_2 &= \xi(x_5'\sin\varphi_5 + y_5'\cos\varphi_5) \end{aligned} \tag{4}$$

where:

$$x_2' = \frac{\mathrm{d}\,x_2}{\mathrm{d}\,\lambda};\; y_2' = \frac{\mathrm{d}\,y_2}{\mathrm{d}\,\lambda};\; x_5' = \frac{\mathrm{d}\,x_5}{\mathrm{d}\,\gamma};\; y_5' = \frac{\mathrm{d}\,y_5}{\mathrm{d}\,\gamma}. \tag{5}$$

After eliminating the parameter ξ the next equality is obtained:

$$(x_2'x_5' + y_2'y_5')\sin(\varphi_2 - \varphi_5) = (x_2'y_5' - y_2'x_5')\cos(\varphi_2 - \varphi_5). \tag{6}$$

Next we will write the contour equations:

$$\begin{aligned}
&\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}\\
&\overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GH} + \overrightarrow{HI} = \overrightarrow{EI}\\
&\overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GJ} + \overrightarrow{JK} = \overrightarrow{EK}\\
&\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BM} = \overrightarrow{OE} + \overrightarrow{EF} + \overrightarrow{FM}
\end{aligned} \tag{7}$$

Measuring the angles made by the elements with the axis OX from the end of the element in the counter clockwise sense, like in figure 3, we obtain the projection of the relations (7) on the reference system OXY axes:

$$\begin{aligned}
&AB\cos\varphi_1 + BC\cos\varphi_2 + CD\cos\varphi_3 = X_D - X_A\\
&AB\sin\varphi_1 + BC\sin\varphi_2 + CD\sin\varphi_3 = Y_D - Y_A\\
&EF\cos\varphi_4 + FG\cos\varphi_5 + GH\cos(\varphi_6 - \alpha) + HI\cos\varphi_7 = X_I - X_E\\
&EF\sin\varphi_4 + FG\sin\varphi_5 + GH\sin(\varphi_6 - \alpha) + HI\sin\varphi_7 = Y_I - Y_E\\
&EF\cos\varphi_4 + FG\cos\varphi_5 + GJ\cos\varphi_6 + JK\cos\varphi_8 = X_K - X_E\\
&EF\sin\varphi_4 + FG\sin\varphi_5 + GJ\sin\varphi_6 + JK\sin\varphi_8 = Y_K - Y_E\\
&X_A + AB\cos\varphi_1 + x_2\cos\varphi_2 - y_2\sin\varphi_2 = X_E + EF\cos\varphi_4 + x_5\cos\varphi_5 - y_5\sin\varphi_5\\
&Y_A + AB\sin\varphi_1 + x_2\sin\varphi_2 + y_2\cos\varphi_2 = Y_E + EF\sin\varphi_4 + x_5\sin\varphi_5 + y_5\cos\varphi_5
\end{aligned} \tag{8}$$

In the relations (8), the unknowns are: φ_2, φ_3, φ_4, φ_5, φ_6, φ_7, φ_8, λ, γ. So, in order to be able to solve the projection equations system, we have to add to the 8 relations (8) the tangent relation (6). We obtain in this way a 9 position functions system:

$$\begin{aligned}
&F_1 = AB\cos\varphi_1 + BC\cos\varphi_2 + CD\cos\varphi_3 - X_D + X_A\\
&F_2 = AB\sin\varphi_1 + BC\sin\varphi_2 + CD\sin\varphi_3 - Y_D + Y_A\\
&F_3 = EF\cos\varphi_4 + FG\cos\varphi_5 + GH\cos(\varphi_6 - \alpha) + HI\cos\varphi_7 - X_I + X_E\\
&F_4 = EF\sin\varphi_4 + FG\sin\varphi_5 + GH\sin(\varphi_6 - \alpha) + HI\sin\varphi_7 - Y_I + Y_E\\
&F_5 = EF\cos\varphi_4 + FG\cos\varphi_5 + GJ\cos\varphi_6 + JK\cos\varphi_8 - X_K + X_E\\
&F_6 = EF\sin\varphi_4 + FG\sin\varphi_5 + GJ\sin\varphi_6 + JK\sin\varphi_8 - Y_K + Y_E\\
&F_7 = X_A + AB\cos\varphi_1 + x_2\cos\varphi_2 - y_2\sin\varphi_2 - X_E - EF\cos\varphi_4 - x_5\cos\varphi_5 + y_5\sin\varphi_5\\
&F_8 = Y_A + AB\sin\varphi_1 + x_2\sin\varphi_2 + y_2\cos\varphi_2 - Y_E - EF\sin\varphi_4 - x_5\sin\varphi_5 - y_5\cos\varphi_5\\
&F_9 = (x_2'x_5' + y_2'y_5')\sin(\varphi_2 - \varphi_5) - (x_2'y_5' - y_2'x_5')\cos(\varphi_2 - \varphi_5)
\end{aligned} \tag{9}$$

If one notates with $\{F\}$ the vector of the nine position functions:

$$\{F\} = \begin{pmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 & F_8 & F_9 \end{pmatrix}^T \tag{10}$$

and with $\{\Phi\}$ the unknowns vector

$$\{\Phi\} = \begin{pmatrix} \varphi_2 & \varphi_3 & \varphi_4 & \varphi_5 & \varphi_6 & \varphi_7 & \varphi_8 & \lambda & \gamma \end{pmatrix}^T \tag{11}$$

one obtains the vectorial equation

$$\{F\} = 0 \tag{12}$$

The solution of the system (12) can be found out using the iterative Newton-Raphson method. To initiate the iterative process it is necessary for us to know the approximate values of the unknowns' vector (11) which is denoted with $\{\Phi^0\}$

$$\{\Phi^0\} = \left(\varphi_2^0 \quad \varphi_3^0 \quad \varphi_4^0 \quad \varphi_5^0 \quad \varphi_6^0 \quad \varphi_7^0 \quad \varphi_8^0 \quad \lambda^0 \quad \gamma^0\right)^T . \tag{13}$$

This is determined by using a graphic method or an assisted graphic one.
The Jacobean $[W]$ of the system (12) is

$$[W] = \begin{bmatrix} \frac{\partial F_1}{\partial \varphi_2} & \frac{\partial F_1}{\partial \varphi_3} & \frac{\partial F_1}{\partial \varphi_4} & \frac{\partial F_1}{\partial \varphi_5} & \frac{\partial F_1}{\partial \varphi_6} & \frac{\partial F_1}{\partial \varphi_7} & \frac{\partial F_1}{\partial \varphi_8} & \frac{\partial F_1}{\partial \lambda} & \frac{\partial F_1}{\partial \gamma} \\ \frac{\partial F_2}{\partial \varphi_2} & \frac{\partial F_2}{\partial \varphi_3} & \frac{\partial F_2}{\partial \varphi_4} & \frac{\partial F_2}{\partial \varphi_5} & \frac{\partial F_2}{\partial \varphi_6} & \frac{\partial F_2}{\partial \varphi_7} & \frac{\partial F_2}{\partial \varphi_8} & \frac{\partial F_2}{\partial \lambda} & \frac{\partial F_2}{\partial \gamma} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ \frac{\partial F_9}{\partial \varphi_2} & \frac{\partial F_9}{\partial \varphi_3} & \frac{\partial F_9}{\partial \varphi_4} & \frac{\partial F_9}{\partial \varphi_5} & \frac{\partial F_9}{\partial \varphi_6} & \frac{\partial F_9}{\partial \varphi_7} & \frac{\partial F_9}{\partial \varphi_8} & \frac{\partial F_9}{\partial \lambda} & \frac{\partial F_9}{\partial \gamma} \end{bmatrix} . \tag{14}$$

We obtain the solution to the iteration *i* ([1]) with the relation:

$$\{\Phi\}_i = \{\Phi\}_{i-1} - [W]_{i-1}^{-1}\{F\}_{i-1} \tag{15}$$

the iterative process continues until

$$\left|\left(\Phi_j\right)_i - \left(\Phi_j\right)_{i-1}\right| \le \varepsilon;\ j = 1, 2, \ldots, 9 , \tag{16}$$

where with □ was denoted the maximum permissible error of the solution. For a given value of the parameter $\square_1$ we obtain the solution (15) with the right precision. Next step, where the angle φ_1 becomes $\varphi_1 + \Delta\varphi_1$, in this way being study all the interval $[0, 2\pi]$. Usually, the angular step is $\Delta\varphi_1 = 1^\circ$.

VELOCITIES ANALYSIS

In a cinematic analysis problem of the bars mechanisms we determine the velocities of the points that materialize the cinematic couples, the velocities of the gravity centers of the elements, the angular velocities of the elements etc. The literature is very rich in things like that.

The problem of the velocities analysis is not very difficult in comparison to the positional analysis problem, when we usually have to solve a nonlinear equations system.
The absolute velocities of the points *B, C, F, G, H, J* and the angular velocities of the elements ω_2, ω_3, ω_4, ω_5, ω_6, ω_7, ω_8 are obtained deriving the position functions and most of all solving a linear equations system.

In the case that there are also class couples in contact, out of the absolute velocities, it presents a special interest the relative velocities in the contact point, these last ones being usually responsible of the wear process.
This is why next we will have a special attention just for the velocities from the contact point *M* between the cam and the follower.

Taking into account the relations (2) that define the position of the point *M* in the general reference system, we obtain the matrix expression:

$$\begin{bmatrix} X_M \\ Y_M \end{bmatrix} = \begin{bmatrix} X_B \\ Y_B \end{bmatrix} + [R_2] \cdot \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_F \\ Y_F \end{bmatrix} + [R_5] \cdot \begin{bmatrix} x_5 \\ y_5 \end{bmatrix} \tag{17}$$

where:

$$X_B = X_A + AB\cos\varphi_1;\ Y_B = Y_A + AB\sin\varphi_1;\ X_F = X_E + EF\cos\varphi_4;\ Y_F = Y_E + EF\sin\varphi_4;$$

$$[R_2] = \begin{bmatrix} \cos\varphi_2 & -\sin\varphi_2 \\ \sin\varphi_2 & \cos\varphi_2 \end{bmatrix};\ [R_5] = \begin{bmatrix} \cos\varphi_5 & -\sin\varphi_5 \\ \sin\varphi_5 & \cos\varphi_5 \end{bmatrix}; \tag{18}$$

Knowing the angular velocity $\omega_1 = \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t}$ of the element 1, the angular velocity of the cam 2 is:

$$\omega_2 = \frac{\mathrm{d}\varphi_2}{\mathrm{d}t} = \frac{\mathrm{d}\varphi_2}{\mathrm{d}\varphi_1}\omega_1, \tag{19}$$

and of the follower 5 is:

$$\omega_5 = \frac{\mathrm{d}\varphi_5}{\mathrm{d}t} = \frac{\mathrm{d}\varphi_5}{\mathrm{d}\varphi_1}\omega_1. \tag{20}$$

To study the relative movement from the point *M*, we introduce a ring (noted with 3) of zero dimension between the bodies in contact. We note with $\vec{v}_{M_2}$, $\vec{v}_{M_3}$ and $\vec{v}_{M_5}$ the absolute velocities of the points M_2, M_3 and M_5, cu $\vec{v}_{M_3M_2}$, $\vec{v}_{M_3M_5}$ the relative velocities of the point M_3 on the cam, also on the follower (fig. 4).

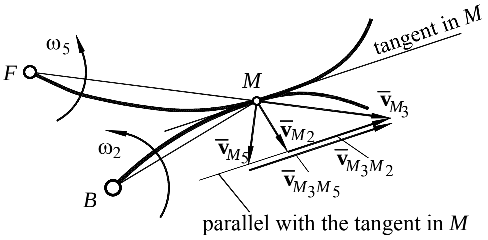

Fig. 4. The absolute and the relative velocities in the contact point *M*.

relative movement between the two profiles is written vectorial:

$$\vec{v}_{M_3} = \vec{v}_{M_2} + \vec{v}_{M_3M_2} = \vec{v}_{M_5} + \vec{v}_{M_3M_5} \tag{21}$$

Deriving in relation to time the relation (17) we obtain

$$\{v_{M_3}\} = \begin{bmatrix} \dot{X}_B \\ \dot{Y}_B \end{bmatrix} + [\dot{R}_2] \cdot \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + [R_2] \cdot \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \dot{X}_F \\ \dot{Y}_F \end{bmatrix} + [\dot{R}_5] \cdot \begin{bmatrix} x_5 \\ y_5 \end{bmatrix} + [R_5] \cdot \begin{bmatrix} \dot{x}_5 \\ \dot{y}_5 \end{bmatrix} \tag{22}$$

where:

$$[\dot{R}_2] = \omega_2 \begin{bmatrix} -\sin\varphi_2 & -\cos\varphi_2 \\ \cos\varphi_2 & -\sin\varphi_2 \end{bmatrix} = \omega_2 \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot [R_2] = \omega_2 \cdot [\Omega] \cdot [R_2] \tag{23}$$

and analogue

$$[\dot{R}_5] = \omega_5 \cdot [\Omega] \cdot [R_5] \tag{24}$$

From the relations (22) are obtained the equalities from below:

$$\begin{aligned} \{v_{M_3}\} &= \omega_1 \begin{bmatrix} \mathrm{d}X_B/\mathrm{d}\varphi_1 \\ \mathrm{d}Y_B/\mathrm{d}\varphi_1 \end{bmatrix} + \omega_2 [\Omega] \cdot [R_2] \cdot \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \omega_2 [R_2] \cdot \begin{bmatrix} \mathrm{d}x_2/\mathrm{d}\varphi_1 \\ \mathrm{d}y_2/\mathrm{d}\varphi_1 \end{bmatrix} = \\ &= \omega_1 \begin{bmatrix} \mathrm{d}X_F/\mathrm{d}\varphi_1 \\ \mathrm{d}Y_F/\mathrm{d}\varphi_1 \end{bmatrix} + \omega_5 [\Omega] \cdot [R_5] \cdot \begin{bmatrix} x_5 \\ y_5 \end{bmatrix} + \omega_5 [R_5] \cdot \begin{bmatrix} \mathrm{d}x_5/\mathrm{d}\varphi_1 \\ \mathrm{d}y_5/\mathrm{d}\varphi_1 \end{bmatrix} \end{aligned} \tag{25}$$

and comparing the result given by the relation (21), using the identification, it results:

$$\{v_{M_2}\}=\omega_1\begin{bmatrix}\mathrm{d}X_B/\mathrm{d}\varphi_1\\ \mathrm{d}Y_B/\mathrm{d}\varphi_1\end{bmatrix}+\omega_2[\Omega]\cdot[R_2]\cdot\begin{bmatrix}x_2\\ y_2\end{bmatrix};\ \{v_{M_3M_2}\}=\omega_2[R_2]\cdot\begin{bmatrix}\mathrm{d}x_2/\mathrm{d}\varphi_1\\ \mathrm{d}y_2/\mathrm{d}\varphi_1\end{bmatrix}$$
$$\{v_{M_5}\}=\omega_1\begin{bmatrix}\mathrm{d}X_F/\mathrm{d}\varphi_1\\ \mathrm{d}Y_F/\mathrm{d}\varphi_1\end{bmatrix}+\omega_5[\Omega]\cdot[R_5]\cdot\begin{bmatrix}x_5\\ y_5\end{bmatrix};\ \{v_{M_3M_5}\}=\omega_5[R_5]\cdot\begin{bmatrix}\mathrm{d}x_5/\mathrm{d}\varphi_1\\ \mathrm{d}y_5/\mathrm{d}\varphi_1\end{bmatrix} \tag{26}$$

ACCELERATIONS ANALYSIS

Like for the velocities, next we will present the relations that define the absolute and the relative accelerations from the contact point M between the cam and the follower.

We note with $\bar{a}_{M_2}$, $\bar{a}_{M_5}$, $\bar{a}_{M_3}$ the absolute accelerations of the points M_2, M_3, M_5 with $\bar{a}^c_{M_3M_2}$, $\bar{a}^c_{M_3M_5}$ the Coriolis accelerations and with $\bar{a}_{M_3M_2}$, $\bar{a}_{M_3M_5}$ the relative accelerations.

In point M is written the vectorial equation

$$\bar{a}_{M_3}=\bar{a}_{M_2}+\bar{a}^c_{M_3M_2}+\bar{a}_{M_3M_2}=\bar{a}_{M_5}+\bar{a}^c_{M_3M_5}+\bar{a}_{M_3M_5} \tag{27}$$

Deriving in relation to time the relation (22) we obtain the equalities:

$$\begin{aligned}\{a_{M_3}\}&=\varepsilon_1\begin{bmatrix}\mathrm{d}X_B/\mathrm{d}\varphi_1\\ \mathrm{d}Y_B/\mathrm{d}\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2X_B/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2Y_B/\mathrm{d}\varphi_1^2\end{bmatrix}+\left[\varepsilon_2[\Omega]\cdot[R_2]-\omega_2^2[R_2]\right]\cdot\begin{bmatrix}x_2\\ y_2\end{bmatrix}+\\&+2\omega_2^2\frac{\mathrm{d}\varphi_2}{\mathrm{d}\varphi_1}[\Omega]\cdot[R_2]\cdot\begin{bmatrix}\mathrm{d}x_2/\mathrm{d}\varphi_1\\ \mathrm{d}y_2/\mathrm{d}\varphi_1\end{bmatrix}+[R_2]\cdot\left[\varepsilon_1\begin{bmatrix}\mathrm{d}x_2/\mathrm{d}\varphi_1\\ \mathrm{d}y_2/d\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2x_2/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2y_2/\mathrm{d}\varphi_1^2\end{bmatrix}\right]=\\&=\varepsilon_1\begin{bmatrix}\mathrm{d}X_F/\mathrm{d}\varphi_1\\ \mathrm{d}Y_F/\mathrm{d}\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2X_F/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2Y_F/\mathrm{d}\varphi_1^2\end{bmatrix}+\left[\varepsilon_5[\Omega]\cdot[R_5]-\omega_5^2[R_5]\right]\cdot\begin{bmatrix}x_5\\ y_5\end{bmatrix}+\\&+2\omega_5^2\frac{\mathrm{d}\varphi_5}{\mathrm{d}\varphi_1}[\Omega]\cdot[R_5]\cdot\begin{bmatrix}\mathrm{d}x_5/\mathrm{d}\varphi_1\\ \mathrm{d}y_5/\mathrm{d}\varphi_1\end{bmatrix}+[R_5]\cdot\left[\varepsilon_1\begin{bmatrix}\mathrm{d}x_5/\mathrm{d}\varphi_1\\ \mathrm{d}y_5/\mathrm{d}\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2x_5/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2y_5/\mathrm{d}\varphi_1^2\end{bmatrix}\right]\end{aligned} \tag{28}$$

and next, comparing with the relation (27), by identification, we obtain the equalities:

$$\begin{aligned}&\{a_{M_2}\}=\varepsilon_1\begin{bmatrix}\mathrm{d}X_B/\mathrm{d}\varphi_1\\ \mathrm{d}Y_B/\mathrm{d}\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2X_B/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2Y_B/\mathrm{d}\varphi_1^2\end{bmatrix}+\left[\varepsilon_2[\Omega]\cdot[R_2]-\omega_2^2[R_2]\right]\cdot\begin{bmatrix}x_2\\ y_2\end{bmatrix};\\&\{a^c_{M_3M_2}\}=2\omega_2^2\frac{\mathrm{d}\varphi_2}{\mathrm{d}\varphi_1}[\Omega]\cdot[R_2]\cdot\begin{bmatrix}\mathrm{d}x_2/\mathrm{d}\varphi_1\\ \mathrm{d}y_2/\mathrm{d}\varphi_1\end{bmatrix};\{a_{M_3M_2}\}=[R_2]\cdot\left[\varepsilon_1\begin{bmatrix}\mathrm{d}x_2/\mathrm{d}\varphi_1\\ \mathrm{d}y_2/d\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2x_2/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2y_2/\mathrm{d}\varphi_1^2\end{bmatrix}\right];\\&\{a_{M_5}\}=\varepsilon_1\begin{bmatrix}\mathrm{d}X_F/\mathrm{d}\varphi_1\\ \mathrm{d}Y_F/\mathrm{d}\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2x_5/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2y_5/\mathrm{d}\varphi_1^2\end{bmatrix}+\left[\varepsilon_5[\Omega]\cdot[R_5]-\omega_5^2[R_5]\right]\cdot\begin{bmatrix}x_5\\ y_5\end{bmatrix};\\&\{a^c_{M_3M_5}\}=2\omega_5^2\frac{\mathrm{d}\varphi_5}{\mathrm{d}\varphi_1}[\Omega]\cdot[R_5]\cdot\begin{bmatrix}\mathrm{d}x_5/\mathrm{d}\varphi_1\\ \mathrm{d}y_5/\mathrm{d}\varphi_1\end{bmatrix};\{a_{M_3M_5}\}=[R_5]\cdot\left[\varepsilon_1\begin{bmatrix}\mathrm{d}x_5/\mathrm{d}\varphi_1\\ \mathrm{d}y_5/\mathrm{d}\varphi_1\end{bmatrix}+\omega_1^2\begin{bmatrix}\mathrm{d}^2x_5/\mathrm{d}\varphi_1^2\\ \mathrm{d}^2y_5/\mathrm{d}\varphi_1^2\end{bmatrix}\right].\end{aligned} \tag{29}$$

CALCULATION ALGORITHM FOR CINEMATIC ANALYSIS

In the case of the multicontour mechanism are known:

- the cam and the follower profile given by the functions $x_2(\lambda), y_2(\lambda)$, $x_5(\gamma), y_5(\gamma)$ and their derivatives of first and second order x_2', y_2', x_5', y_5', x_2'', y_2'', x_5'', y_5'';
- the dimensions l_i, $i=1,2,...,8$ and the approximate positions of the couples from the articulated mechanism;
- the maximum error value ε of determination of the solution, $\varepsilon = 0{,}000001$.

The calculation algorithm is:

- Using the relations (1) ÷ (16) we make the computation program for the positional analysis. This has in its composition two procedures, one to compute the position functions given by the relations (9) and one to compute the jacobian given by the relations (14).
- In a repetitive cycle we give values for the angle φ_1 from one degree to another from 0° to 360° with a constant step of 1°. For each step are determined in a repetitive cycle (18) with the precision $\varepsilon = 10^{-6}$. Calculation formulas are the ones given by the relations: (9), (14), (15).
- The beginning values of the iterative process: φ_2^0, φ_3^0, φ_4^0, φ_5^0, φ_6^0, φ_7^0, φ_8^0 $\lambda^0 = 0,\ \gamma^0 = 0$ are necessary only for $\varphi_1 = 0$, for the next steps the approximate necessary values of the iterative process are exactly the values obtained previously. After the exit from the repetitive cycle we retain the values determined previously as vectors of 360 position.
- In the end we write in a text folder the obtained values.

Based on the analytic relations of determining the velocities and the accelerations, we continue the computation program with two other procedures:

- the numerical procedure of obtaining the reduced velocities and accelerations based on the relations of numerical derivation:

$$\left.\frac{d\varphi_j}{d\varphi_1}\right|_{\varphi_1=\varphi_{1i}} = \frac{\varphi_{j,i+1}-\varphi_{j,i-1}}{2\Delta\varphi_1};\ \left.\frac{d^2\varphi_j}{d\varphi_1^2}\right|_{\varphi_1=\varphi_{1i}} = \frac{\varphi_{j,i+1}-2\varphi_{j,i}+\varphi_{j,i-1}}{(\Delta\varphi_i)^2},\ j=2,\ 5 \tag{30}$$

- the procedure of obtaining the matrix as a result of the multiplying of two matrices;
- the obtaining of the numerical values of the angular velocities:

$$\frac{d\varphi_2}{d\varphi_1},\frac{d\varphi_5}{d\varphi_1},\frac{dx_2}{d\varphi_1},\frac{dy_2}{d\varphi_1},\frac{dx_5}{d\varphi_1},\frac{dy_5}{d\varphi_1},\frac{dX_B}{d\varphi_1},\frac{dY_B}{d\varphi_1},\frac{dX_F}{d\varphi_1},\frac{dY_F}{d\varphi_1}$$

- and those of the angular accelerations:

$$\frac{d^2\varphi_2}{d\varphi_1^2},\frac{d^2\varphi_5}{d\varphi_1^2},\frac{d^2x_2}{d\varphi_1^2},\frac{d^2y_2}{d\varphi_1^2},\frac{d^2x_5}{d\varphi_1^2},\frac{d^2y_5}{d\varphi_1^2},\frac{d^2X_B}{d\varphi_1^2},\frac{d^2Y_B}{d\varphi_1^2},\frac{d^2X_F}{d\varphi_1^2},\frac{d^2Y_F}{d\varphi_1^2};$$

  - the obtaining in a repetitive cycle FOR in which we give values for the angle φ_1 from one degree to another from 0° to 360° with a constant step of 1° of the numerical values of the velocities and accelerations with the relations (26) and (29);
  - the writing into two text folders of the obtained values,
  - the graphic retrieval of the numerical values.

CONCLUSIONS

The studied mechanisms are often found in the composure of systems that allow changing the height of heat engines valves. Depending on the configuration, each mechanism usually has its own method of cinematic analysis. A general analysis method was obtained by choosing the plan parallel movement for the cam and follower, for the cinematic analysis of multicontour mechanisms with a cam-follower mechanism.

The cinematic analysis method presented in the paper, is based on the contour projecting method and combines the specific cinematic analysis of the cam mechanisms with the cinematic analysis of the plane bars mechanisms. Solving the nonlinear position equations system is done with the iterative Newton-Raphson method, and the determination of the velocities and accelerations is done with numerical methods of derivation.

The method is general and it can be applied to the simple bars mechanisms and also to the complex cam mechanisms. Using the presented algorithm we can realise a computation program by which we obtain numerical results. The algorithm can be completed with the computation relations of the linear and angular velocities and accelerations of the elements. A matrix in which we work allows an easy implementation in any environmental programming.

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