OPTIMAL SYNTHESIS OF A HARBOUR CRANE

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Abstract: *Optimal synthesis of mechanisms is a repeated analysis for a random determined mechanism and finding of the best possible one so that it could meet technological requirements, and it is most often used in dimensional synthesis, which implies determination of elements of the given mechanism (lengths, angles, coordinates) necessary for creation of the mechanism in the direction of desired motion. Dimensional synthesis using analytical techniques has been found to be very useful in mechanism design. A good implementation of this method is in obtaining an initial guess or starting point for optimal synthesis procedures especially in path generation problems. This paper represents the synthesis of a four-bar mechanism, representing the scheme of a harbour crane, as a coupler point to describe a straight, horizontal line.*

Keywords: harbour crane, optimal sysntesis; Pattern Search algorithm.

INTRODUCTION

There are two common requirements in kinematic synthesis of mechanisms: path generation and motion generation. In dimensional synthesis there are two approaches: synthesis of precision points and approximate or optimal synthesis. Precision point synthesis implies that the point on the coupler plane passes through a certain number of desired (exact) points, but without the possibility of controlling a structural error on a path out of those points. Precision point synthesis is restricted by the number of points which must be equal to the number of independent parameters defined by the mechanism [1]. The maximum number of points for a four-bar linkage is nine. If the number of equations generated by the number of exact points is smaller than the number of projected variables, then there is a selection of free variables, so that the problem of synthesis does not have a single-valued solution [2]. When the number of precision points increases, the problem of precision point synthesis becomes very nonlinear and extremely difficult for solving, and the mechanism obtained by this type of synthesis is in most cases useless: because dimensions of the mechanism members are in disproportion, or the obtained solutions are in the form of complex numbers so there is no mechanism. The maximum number of precision points on the path of the coupler in a four-bar-linkage is nine in uncoordinated motion [3].

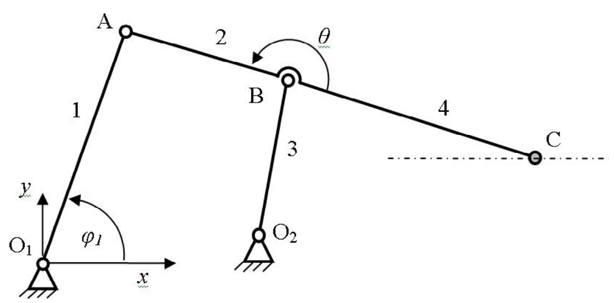

Figure 1. Four-bar linkage of harbour crane

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Kinematic synthesis, in general, implies the development of methods of computational or graphical construction, that implicitly determine the proper dimensions for the synthesized mechanism. Typically, a design process is a sequence of decisions, each of which must be evaluated and altered as necessary. The design engineer brings to this process his experience from past successful designs. The ultimate goal of the design process is the discovery of the optimum solution for a given design situation. With the development of powerful numerical analysis tools, it is becoming increasingly clear that the traditional graphical techniques can be supplemented and sometimes completely replaced by computational methods [4].

A harbour crane as depicted in fig. 1 will be considered. It will be assumed that the four-bar linkage is given and that the point C of the coupler plane must be searched that generates the (approximated) horizontal line.

EQUATIONS OF MOTION

The coordinates of joint A are expressed in terms of the coordinates of joint O_1 and the relative orientation of link 1. Its coordinates are determined using following equations

$$x_A = x_{O1} + l_1 \cos\varphi_1 \tag{1}$$

$$y_A = y_{O1} + l_1 \sin\varphi_1 \tag{2}$$

For the joint B on the links 2 and 3 can be writing

$$x_B = x_A + l_2 \cos\varphi_2 = x_{O2} + l_3 \cos\varphi_3 \tag{3}$$

$$y_B = y_A + l_2 \sin\varphi_2 = x_{O2} + l_3 \sin\varphi_3 \tag{4}$$

where φ_2 and φ_3 are the relative orientations of links 2 and 3 with respect to axis ox of the Cartesian reference frame *Oxy*.

By eliminating φ_3 combining equation (3) with (4) and summarizing we have

$$[(x_A - x_{O2}) + l_2 \cos\varphi_2]^2 + [(y_A - y_{O2}) + l_2 \sin\varphi_2]^2 = l_3^2 \tag{5}$$

or

$$A\cos\varphi_2 + B\sin\varphi_2 + C = 0 \tag{6}$$

Where

$$A = 2l_2(x_A - x_{O2}) \tag{7}$$

$$B = 2l_2(y_A - y_{O2}) \tag{8}$$

$$C = (x_A - x_{O2})^2 + (y_A - y_{O2})^2 + l_2^2 - l_3^2 \tag{9}$$

With notation

$$T = \tan\frac{\varphi_2}{2} \tag{10}$$

equation (6) can be writing as

$$(C - A)T^2 + 2BT + C + A = 0 \tag{11}$$

and is obtained

$$T = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{C - A} \tag{12}$$

respective

$$\varphi_2 = 2\arctan T \tag{13}$$

$$\varphi_4 = \varphi_2 - (\pi + \theta) \tag{14}$$

Similarly, for link 4 and coupler point C can write relations

$$x_C = x_B + l_4 \cos\varphi_4 \tag{15}$$

$$y_C = y_B + l_4 \sin\varphi_4 \tag{16}$$

DETERMINATION OF THE OPTIMAL CONFIGURATION

To find the optimal configuration of the linkage mechanism has been used Matlab program with optimization Pattern Search algorithm [5], [6], [7].

The Pattern Search algorithm uses the Augmented Lagrangian Pattern Search (ALPS) algorithm to solve nonlinear constraint problems. The optimization problem solved by the ALPS algorithm is

$$\min_x f(x) \tag{17}$$

such that

$$c_i(x) \le 0\ ,\ i = 1...m \tag{18}$$

$$ceq_i(x) = 0\ ;\ i = m+1...m_t \tag{19}$$

$$A \cdot x \le b \tag{20}$$

$$Aeq \cdot x \le beq \tag{21}$$

$$lb \le x \le ub \tag{22}$$

where $c(x)$ represents the nonlinear inequality constraints, $ceq(x)$ represents the equality constraints, m is the number of nonlinear inequality constraints, and m_t is the total number of nonlinear constraints.

The ALPS algorithm attempts to solve a nonlinear optimization problem with nonlinear constraints, linear constraints, and bounds. In this approach, bounds and linear constraints are handled separately from nonlinear constraints. A subproblem is formulated by combining the objective function and nonlinear constraint function using the Lagrangian and the penalty parameters. A sequence of such optimization problems are approximately minimized using a pattern search algorithm such that the linear constraints and bounds are satisfied. Each subproblem solution represents one iteration. The number of function evaluations per iteration is therefore much higher when using nonlinear constraints than otherwise [8].

The pattern search minimizes a sequence of subproblems, each of which is an approximation of the original problem. Each subproblem has a fixed value of λ, s, and ρ. When the subproblem is minimized to a required accuracy and satisfies feasibility conditions, the Lagrangian estimates are updated. Otherwise, the penalty parameter is increased by a penalty factor. This results in a new subproblem formulation and minimization problem [9]. These steps are repeated until the stopping criteria are met. Each subproblem solution represents one iteration. The number of function evaluations per iteration is therefore much higher when using nonlinear constraints than otherwise [10]. The imposed (desired) trajectory of coupler-point C is indicated in Table 1.

Table 1. Imposed (desired) trajectory of coupler-point.

$\varphi_1 [^0]$	45	55	65	75	85	95	105	115	125	135
$x_{C-dez}[m]$	6.7	7.03	7.36	7.69	8.02	8.35	8.68	9.01	9.34	9.67
$y_{C-dez}[m]$	1	1	1	1	1	1	1	1	1	1

The linear and nonlinear inequality constrains can be write as form

$$\sqrt{(x_{O2} - x_{O1})^2 + (y_{O2} - y_{O1})^2} + l_1 \le l_2 + l_3 \tag{17}$$

$$\sqrt{(x_{O2} - x_{O1})^2 + (y_{O2} - y_{O1})^2} - l_1 \ge l_3 - l_2 \ge 0 \tag{18}$$

$$-10 \le x_{O1} \le 10 \tag{19}$$

$$-10 \le x_{O2} \le 10 \tag{20}$$

$$-10 \le y_{O2} \le 10 \tag{21}$$

$$0 \le l_i \le 11;\ i = 1...4 \tag{22}$$

$$0 \le \theta \le 2\pi \tag{23}$$

The objective function can be write as form

$$f(x) = \frac{1}{\varphi_{1-\max} - \varphi_{1-\min}} \int_{\varphi_{1-\min}}^{\varphi_{1-\max}} \left[(x_C - x_{C-dez})^2 + (y_C - y_{C-dez})^2 \right] d\varphi \tag{24}$$

where $\varphi_{1-\min} = 45^0; \varphi_{1-\max} = 135^0$.

The vector x of unknown parameters is of the form

$$x = [x_{O1}, x_{O2}, y_{O2}, l_1, l_2, l_3, l_4, \theta]; \tag{25}$$

and its initial value is

$$x_0 = [0,4,0,1,3,3,5, pi/6] \tag{26}$$

Based on the above by Optimization Tool of Matlab and Pattern Search solver are obtained the following results indicated in Table 2 and Figure 2 and Figure 3.

Table 2. Results / Matlab Optmization Tool.

```
x =

    3.5410
   -1.8210
    5.6950
    1.9740
    7.9710
   10.0540
    4.6590
    3.8460
```

```
                                    max
  Iter    f-count        f(x)      constraint    MeshSize        Method
    0        1         33.9191          0            1
    1     3250         2.85313          0        0.009772    Update multipliers
    2     6583        0.256803          0        0.000955    Update multipliers
    3    10460        0.217153          0       9.333e-05    Update multipliers
    4    16000        0.189632          0        9.12e-06    Update multipliers
Maximum number of function evaluations exceeded: increase options.MaxFunEvals.
```

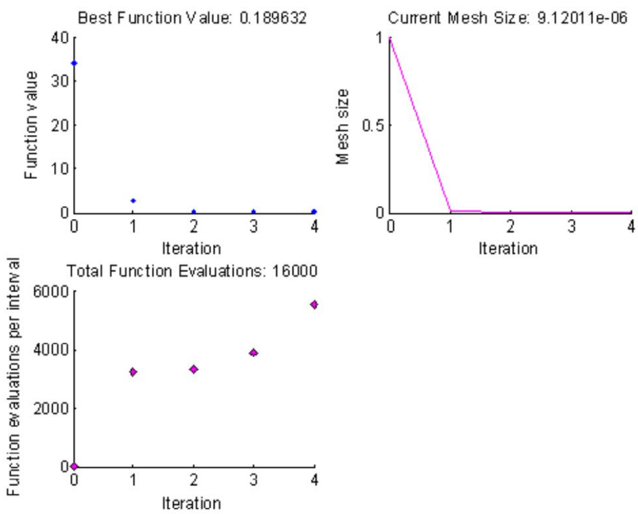
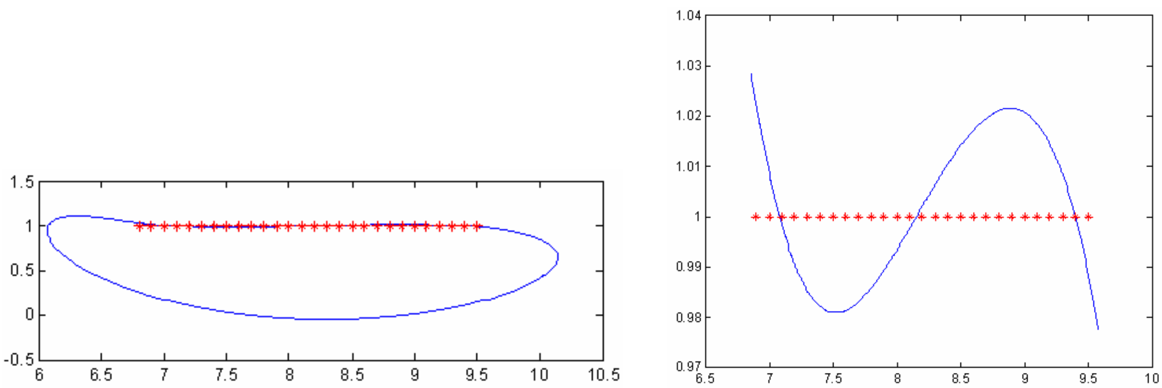


Figure 2. Pattern Search optimization diagrams.



a) b)

Figure 3. Desired versus optimized Coupler point diagrams.

CONCLUSION

As can be seen in the figure above, between required and obtained values by optimization procedure is a very good coincidence. The approximation error, that is the minimum value of the objective function, is 0.189632. Accordance with the procedure described above can be solved with success many of the practical problems of synthesis of planar mechanisms. Can be taken into consideration a multitude of solutions in a short time and optimum solutions results have a high degree of accuracy.

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