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COMPARISON BETWEEN TWO TYPES OF TRIPOD JOINTS

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Abstract: *In the present paper we make a comparison between two types of tripod kinematic pairs. The first type is characterized by the fact that the contact takes place on three straight lines, while on the second type the contact takes place on three identical curves. For each case we determine the equations of the projection curve and in a numerical case we perform a comparison between the results.*

Keywords: tripod kinematic pair, projection curve

INTRODUCTION

We denote by α_i, β_i, and γ_i, $i = 1, 2.3$, the components of the rotational matrix and by $[\mathbf{A}]$, $[\boldsymbol{\psi}]$, $[\boldsymbol{\gamma}]$, $[\boldsymbol{\varphi}]$ the matrices

$$[\mathbf{T}] = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix},\ [\boldsymbol{\psi}] = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix},\ [\boldsymbol{\gamma}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix},$$
$$[\boldsymbol{\varphi}] = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

It results the matrix relation [1]

$$[\mathbf{T}] = [\boldsymbol{\psi}][\boldsymbol{\gamma}][\boldsymbol{\varphi}], \quad (2)$$

with the aid of which we get the expressions of the director cosines as function of Euler's angles.

THE TYPE ONE TRIPOD KINEMATIC PAIR

The type one tripod kinematic pair is presented in Fig. 1.
We denote by B_j, $j = 1, 2, 3$, the points at which the straight lines (Γ_j), equidistant and parallel to the axis O_1z_1, intersect the perpendicular plan to the axis O_1z_1 and passing through the point O_1, and we consider that the axis O_1z_1 is situated in the direction of the straight line O_1B_1 (Fig. 1).
We also make the notations

$$O_jB_j = r,\ B_jA_j = \lambda_j,\ O_2A_j = \mu_j,\ j = 1, 2, 3, \quad (3)$$

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$$\delta_j = \frac{2\pi}{3}(j-1),\ j = 1, 2, 3. \tag{4}$$

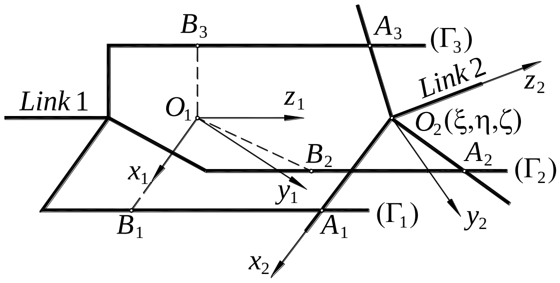


Figure 1. The type one tripod kinematic pair

Let x_{1j}, y_{1j}, z_{1j}, and x_{2j}, y_{2j}, z_{2j} be the coordinates of the point A_j, $j = 1, 2, 3$, relative to the reference systems $O_1x_1y_1z_1$, $O_2x_2y_2z_2$, respectively.
Taking into account the expressions

$$x_{1j} = r\cos\delta_j,\ y_{1j} = r\sin\delta_j,\ z_{1j} = \lambda_j, \tag{5}$$

$$x_{2j} = \mu_j\cos\delta_j,\ y_{2j} = \mu_j\sin\delta_j,\ z_{2j} = 0 \tag{6}$$

and the matrix relation

$$\begin{bmatrix} x_{1j} \\ y_{1j} \\ z_{1j} \end{bmatrix} = \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} x_{2j} \\ y_{2j} \\ z_{2j} \end{bmatrix}, \tag{7}$$

by eliminating the parameter μ_j, we obtain the equalities

$$\begin{matrix} \xi(\beta_1\cos\delta_j + \beta_2\sin\delta_j) - \eta(\alpha_1\cos\delta_j + \alpha_2\sin\delta_j) \\ -r[\beta_1\cos^2\delta_j + (\beta_2 - \alpha_1)\sin\delta_j\cos\delta_j - \alpha_2\sin\delta_j] = 0, \end{matrix},\ j = 1, 2, 3. \tag{8}$$

Adding the relations (8) and taking into account the well known trigonometric relations

$$\sum_{i=1}^{3}\cos\delta_j = \sum_{i=1}^{3}\sin\delta_j = \sum_{i=1}^{3}\sin\delta_j\cos\delta_j = 0, \tag{9}$$

$$\sum_{i=1}^{3}\cos^2\delta_j = \sum_{i=1}^{3}\sin^2\delta_j = \frac{3}{2}, \tag{10}$$

we obtain the essential condition of the first type tripod kinematic pair [1]

$$\beta_1 = \alpha_2; \tag{11}$$

using the Euler angles, the previous relation takes the form

$$\psi = -\varphi. \tag{12}$$

Using the relations (11), (12), (1), (2), (7), and (8), we also obtain the expressions

$$\xi = \frac{r(1-\cos\gamma)}{2\cos\gamma}[\cos(3\varphi)\cos\varphi + \cos\gamma\sin(3\varphi)\sin\varphi], \tag{13}$$

$$\eta = \frac{r(1-\cos\gamma)}{2\cos\gamma}\left[-\cos(3\varphi)\sin\varphi + \cos\gamma\sin(3\varphi)\cos\varphi\right], \tag{14}$$

$$\mu_j = \frac{r\cos\delta_j - \xi}{\alpha_1\cos\delta_j + \alpha_2\sin\delta_j}, \tag{15}$$

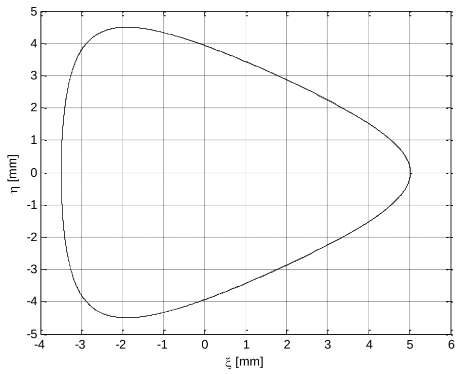
$$\lambda_j = \zeta + (\gamma_1\cos\delta_j + \gamma_2\sin\delta_j)\mu_j, \tag{16}$$

in which ζ, φ, and γ are independent parameters.

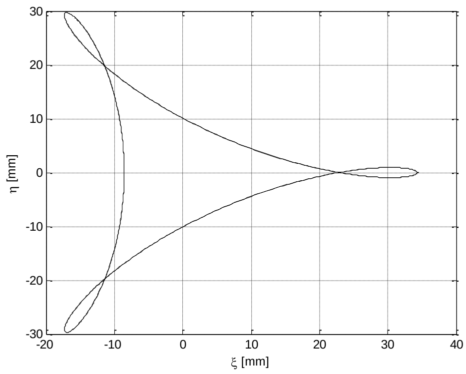
In the case of the tripod joints used in practice, the variation of the angle γ during the joint's work is a small one; hence, we may realize a good image of the joint's work by considering that the angle γ is constant.

In this situation, when $\gamma = \text{const}$, the projection of the curve described by the point O_2 onto the plan $O_1x_1y_1$ (Fig. 1) is curve given by the equations (13) and (14).

For instance, when $\gamma = 46^0$, $r = 22.85\,\text{mm}$, the curve has the representation given in Figure 2 *a)*, while in the case in which $\gamma = 75.5224878^0$ ($\cos\gamma = 0.25$) and the parameter r remains constant, the curve is captured in Figure 2 *b)*.



a)



b)

Figure 2. The projection of the curve described by the point O_2 onto the plan $O_1x_1y_1$
a) case $\gamma = 46^0$, $r = 22.85\,\text{mm}$; *b)* case $\gamma = 75.5224878^0$ ($\cos\gamma = 0.25$), $r = 22.85\,\text{mm}$

From the analysis of the results presented above, it follows that when $\gamma > \arccos\left(\frac{1}{3}\right)$, the point O_2 comes out of the circle of radius r; since this condition is difficult to be obtained in practice, it results that a first limitation for the angle γ is given by the relation

$$\gamma < \arccos\left(\frac{1}{3}\right) = 70.52288^0. \tag{17}$$

Further on, we calculate the distance s between the points O_2 and O_1 (Fig. 1), and we get the following expression

$$s = \frac{r(1-\cos\gamma)}{2\cos\gamma}\sqrt{\cos^2(3\varphi) + \cos^2\gamma\sin^2(3\varphi) + \zeta^2}, \tag{18}$$

the minimum variation of which being when $\zeta = 0$.

It follows that for $\gamma \neq 0$, one cannot make a type one tripod joint for which the axes O_2z_2 and O_1z_1 are concurrent, but one can make a pseudo-angular tripod joint for which the distance s

between the points O_1 and O_2 has a minimal variation for $\zeta = 0$ and, in addition, it verifies the relation

$$\frac{r(1-\cos\gamma)}{2\cos\gamma} \le s \le \frac{r(1-\cos\gamma)}{2}. \tag{19}$$

From the constructive point of view, the condition of angular joint ($\zeta = 0$) is fulfilled with the aid of a bilateral kinematic pair of sphere-plan type (Figure 3).

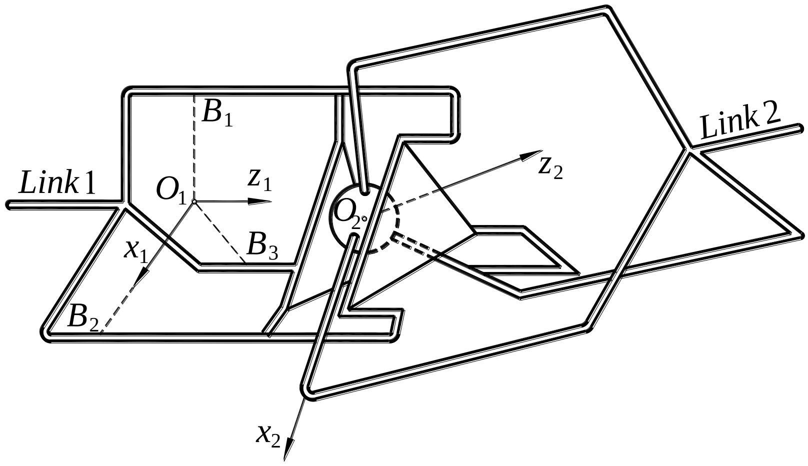


Figure 3. Angular first type tripod kinematic pair

THE TYPE TWO TRIPOD KINEMATIC PAIR

The curves (Γ_j), $j = 1, 2, 3$, are symmetrically situated in space; it follows that if the curve (Γ_1) has the equations

$$x_1 = f(z_1),\ y_1 = 0, \tag{20}$$

in the system $O_1x_1y_1z_1$, then the curves (Γ_j), $j = 1, 2, 3$, will be described by the equations

$$x_1 = f(z_1)\cos\delta_j,\ y_1 = f(z_1)\sin\delta_j,\ j = 1, 2, 3. \tag{21}$$

Similarly, for the straight lines (Δ_j), $j = 1, 2, 3$, one obtains the following equations

$$y_2 = x_2 \tan\delta_j,\ z_2 = 0. \tag{22}$$

From the above discussion it results that, in the system $O_1x_1y_1z_1$, the coordinates of the point A_j, $j = 1, 2, 3$, are

$$x_{1j} = f(z_{1j})\cos\delta_j,\ y_{1j} = f(z_{1j})\sin\delta_j,\ z_{1j} = z_{1j}, \tag{23}$$

while in the system $O_2x_2y_2z_2$ the coordinates are

$$x_{2j} = x_{2j},\ y_{2j} = x_{2j}\tan\delta_j,\ z_{2j} = 0; \tag{24}$$

taking into account the matrix relation of transformation

$$\begin{bmatrix} x_{1j} \\ y_{1j} \\ z_{1j} \end{bmatrix} = \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} x_{2j} \\ x_{2j}\tan\delta_j \\ 0 \end{bmatrix}, \tag{25}$$

one gets the following expressions

$$f(z_{1j})\cos\delta_j = \xi + (\alpha_1 + \alpha_2 \tan\delta_j)x_{2j}\,,\; f(z_{1j})\sin\delta_j = \eta + (\beta_1 + \beta_2 \tan\delta_j)x_{2j}\,,$$
$$z_{1j} = \zeta + (\gamma_1 + \gamma_2 \tan\delta_j)x_{2j}\,. \tag{26}$$

Eliminating the parameters x_{2j} and z_{1j} from the relations (26) and denoting

$$U_j = \xi(\beta_1 \cos\delta_j + \beta_2 \sin\delta_j) - \eta(\alpha_1 \cos\delta_j + \alpha_2 \sin\delta_j), \tag{27}$$

$$V_j = (\beta_2 - \alpha_1)\sin\delta_j \cos\delta_j + \beta_1 \cos^2\delta_j - \alpha_2 \sin^2\delta_j\,, \tag{28}$$

$$W_j = (\xi \sin\delta_j - \eta\cos\delta_j)(\gamma_1 \cos\delta_j + \gamma_2 \sin\delta_j) \tag{29}$$

we obtain the relations

$$V_j f\left(\xi + \frac{W_j}{V_j}\right) - U_j = 0\,,\; j = 1, 2, 3. \tag{30}$$

The three relations (30) represent the equations of the type two tripod kinematic pairs.
If the curves (Γ_j), $j = 1, 2, 3$, are straight lines, then $f(z_1) = r$, and the relations (30) may be written in the form

$$rV_j - U_j = 0\,,\; j = 1, 2, 3; \tag{31}$$

by addition of these relations, one finds again the condition (11).
In the case when the joint is an angular one, then, putting $\zeta = 0$ and using Euler's angles, for imposed values for the angle γ, from the system (30), one may determine the parameters ψ, ξ, and η as functions of the angle φ.
In the practical cases, the function f may be defined as a third degree polynomial function in the form

$$f(z_1) = r + \lambda_1^* z_1 + \lambda_2^* z_1^2 + \lambda_3^* z_1^3 \tag{32}$$

and the relations (30) become

$$-U_j V_j^2 + rV_j^3 + \lambda_1^* V_j^2 W_j + \lambda_2^* V_j W_j^2 + \lambda_3^* W_j^3 = 0\,. \tag{33}$$

In the numerical cases, the system of equations (33) may be solved with the aid of the Newton-Raphson method [2] and using a calculation program.
Considering the numerical data $\gamma = 46^0$, $r = 22.85\,\text{mm}, \lambda_1^* = -0.0124$, $\lambda_2^* = -0.00083$, $\lambda_3^* = -0.000018$, the parameters ξ, η, and ψ are determined as functions of the angle φ, as well as the coordinates of the point O_2 in the two reference systems. These results are compared with those ones obtained for the type one tripod kinematic pair for which γ and $r = 22.85\,\text{mm}$; the corresponding diagrams are drawn in Figure 4 and Figure 5.
We find that that for the curve of the type two tripod kinematic pair, drawn in Figure. 4, the distance $\rho = \sqrt{\xi^2 + \eta^2}$ has closer extreme values than in the case of the curve corresponding to the type one tripod kinematic pair.
Denoting by $\Delta\psi$ the difference between the values of the angle ψ which correspond to the type two, and type one tripod kinematic pairs, respectively, one obtains the graphic representation in Figure 5, from which it follows that this difference is a periodical one and, in addition, it has a small maximum value.

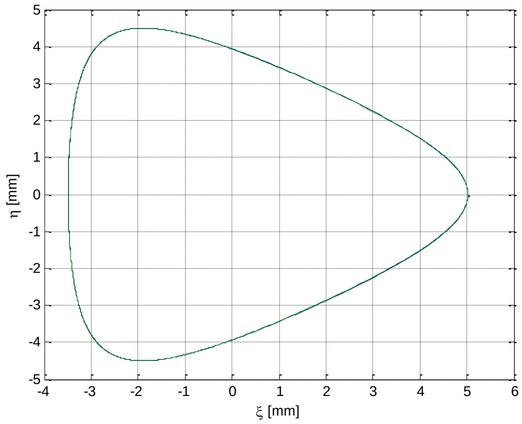


Figure 4. The projection of the curve described by the point O_2 onto the plan $O_1x_1y_1$ in the case of first type tripod kinematic pair (green) and second type kinematic pair (blue).

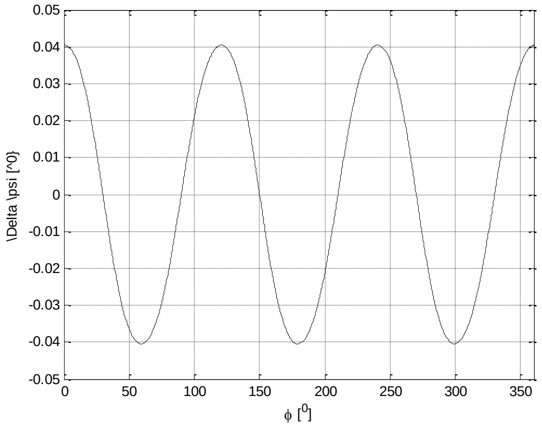


Figure 5. The difference between the angles ψ corresponding to the two types of kinematic pairs.

CONCLUSIONS

In this paper we performed a comparison between two types of tripod kinematic pairs at which the contact takes place on straight lines or some curves. The essential conditions are obtained in each case. In the first situation the curve may have different shapes, all of them being hypocycloids. The discuss may be generalized for other types of tripod kinematic pairs.

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