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The determination of the mechanical friction work in a cam – follower couple of complex cam mechanisms. Part I - theoretical aspects

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Abstract – In this paper we are presenting theoretical aspects regarding the determination of the mechanical friction work presented in a cam-follower couple, in the case where the follower has a plane parallel movement. An articulated quadrilateral mechanism it is cinematically and dynamically studied, where the connecting rod is actioned by a rotation cam. For showing the relative movement of the contact point on the cam and on the follower, a zero-dimension ring is inserted between the cam and the follower. The expression of the mechanical work is obtained after solving the cinematic and dynamical analysis of the complex rotation cam mechanism. The value of the mechanical work is obtained by integration. The conclusions are presented in the end of the paper and it is mentioned that in a future paper, a numerical application will be made for illustrating the calculation algorithm.

Keywords – cam, follower, positions, velocities, reaction forces, mechanical work

1. Introduction

Cam mechanisms are often used in car manufacturing in different industrial domains. The literature is rich with information regarding the arrangement of complex cam mechanisms, cinematic analysis and with the synthesis of these mechanisms.
The mechanisms made only from a cam and a follower are called simple cam mechanisms. By amplifying a simple cam mechanism with a structural group (dyad, triad, tetrad), a complex cam mechanism is obtained. These complex cam mechanisms are used in the construction of different cars or mechanisms used in the construction of distribution systems in heat engines. In [1] we are presenting many of these solutions for these mechanisms. A sorting of these, considering their functional role, it is done in [7].
The cinematic and dynamic analysis of complex cam mechanisms is presented in detail in [2], [3] and [6]. The cinematic analysis of complex cam mechanisms is addressed in the papers [5] and [8]. In the present paper we are proposing to determine the mechanical friction work from the cam-follower couple, of complex cam mechanisms.

2. Problem formulation

The basic mechanical friction work from the cinematic couple is given by the relation:
$$\mathrm{d}L = F_f v_r \,\mathrm{d}t = \mu N v_r \,\mathrm{d}t \,. \tag{1}$$
In (1) it was noted with F_f the sliding friction force, with N the normal reaction, with μ the value of the sliding friction force and with v_r the relative velocity between the two bodies in contact.

In the case of a mechanism, for obtaining the value of the mechanical friction work, we are integrating the equation (1) on a cinematic cycle. Considering the angular velocity of the motor element is constant $\omega = \frac{\mathrm{d}\varphi}{\mathrm{d}t} ct.$, the following expression is obtained:

$$L = \frac{\mu}{\omega} \int_0^{2\pi} N v_r \, \mathrm{d}\varphi . \tag{2}$$

3. Cinematic aspects

For determining the mechanical friction work we will have to determine the relative velocity in the contact point of the cam and the follower.

In the case of complex cam mechanisms with a mobile cam, the follower can have a translation movement, a rotation movement or a plane parallel movement. We will analyze the case where the follower has a plane parallel movement, the other cases being particular cases of this one.

In fig. 1, the cam actuates the rod *AB* of an articulated quadrilateral mechanisms *OABC*, the cams groove 2 (attached to the connecting rod) is being circular, of radius *R*. The contact between the cam and the follower is done by the spring 5.

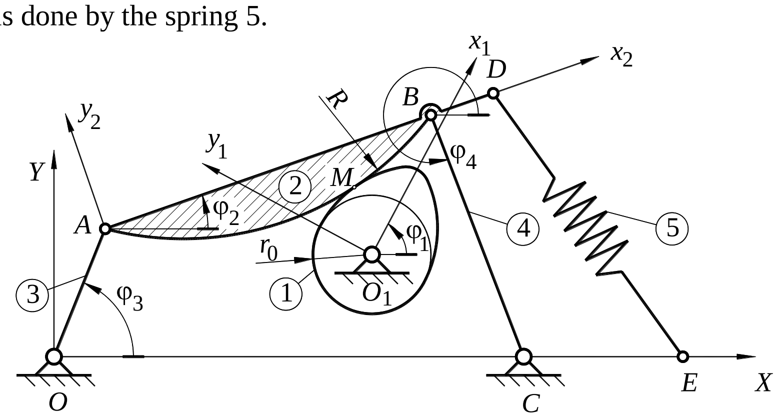

Figure 1. Complex cam mechanism with rotation cam and curved follower in plane parallel movement.

For studying the relative displacement between the cam and the follower we are introducing in *M*, a zero-dimension ring 6. There are highlighted the relative movements on the cam's and follower's groove. We are noting with $\vec{v}_{M_1}$, $\vec{v}_{M_2}$ şi $\vec{v}_{M_6}$ the absolute velocities of points M_1, M_2 and M_6, with $\vec{v}_{M_1M_6}$, $\vec{v}_{M_2M_6}$ the relative velocity of point M_6 on the cam, and on the follower (fig. 2).

From the study of relative movement between the cam and the follower, the following relations are obtained:

$$\vec{v}_{M_6} = \vec{v}_{M_1} + \vec{v}_{M_6M_1} = \vec{v}_{M_2} + \vec{v}_{M_6M_2} . \tag{3}$$

In fig. 2 there are presented the relative velocities of the contact point *M* and the cam's groove 1 and follower's groove 2.

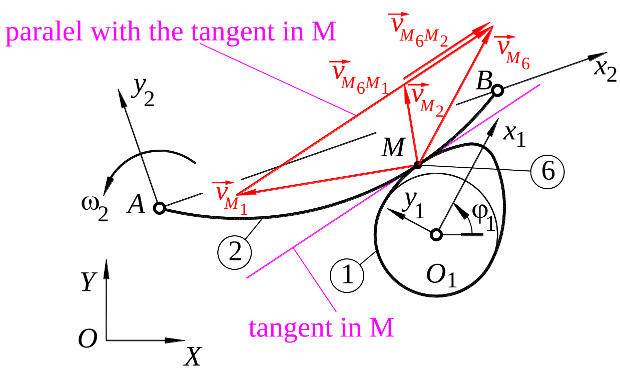

Figure 2. Velocities polygon in point *M*.

For obtaining the velocities in the point M in the $x_1O_1y_1$ reference system and respectively x_2Ay_2, the following relations between the coordinates of point M are written:

$$\begin{cases} x_M = x_{O_1} + x_1\cos\varphi_1 - y_1\sin\varphi_1 = x_A + x_2\cos\varphi_2 - y_2\sin\varphi_2 \\ y_M = y_{O_1} + x_1\sin\varphi_1 + y_1\cos\varphi_1 = y_A + x_2\sin\varphi_2 + y_2\cos\varphi_2 \end{cases}, \tag{4}$$

The relations (4) are written in the matrix form:

$$\begin{bmatrix} x_M \\ y_M \end{bmatrix} = \begin{bmatrix} x_{O_1} \\ y_{O_1} \end{bmatrix} + [R_1]\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \text{ where } [R_1] = \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{bmatrix}, \tag{5}$$

respectively

$$\begin{bmatrix} x_M \\ y_M \end{bmatrix} = \begin{bmatrix} x_A \\ y_A \end{bmatrix} + [R_2]\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \text{ where } [R_2] = \begin{bmatrix} \cos\varphi_2 & -\sin\varphi_2 \\ \sin\varphi_2 & \cos\varphi_2 \end{bmatrix}. \tag{6}$$

In the previous relations:

$$\begin{aligned} & x_1 = x_1(\lambda_1),\ y_1 = y_1(\lambda_1), \\ & x_2 = x_2(\lambda_2),\ y_2 = y_2(\lambda_2), \end{aligned} \tag{7}$$

there are defined the parametric coordinates of the cam's groove, also of the follower, in their own reference system.

By knowing the angular velocity $\omega_1 = \dfrac{\mathrm{d}\varphi_1}{\mathrm{d}t}$ of element 1, the angular velocity of cam 2 is:

$$\omega_2 = \frac{\mathrm{d}\varphi_2}{\mathrm{d}t} = \frac{\mathrm{d}\varphi_2}{\mathrm{d}\varphi_1}\omega_1 . \tag{8}$$

Deriving the relations (5) and (6) we obtain the vector relations of the velocities:

$$\{v_{M_6}\} = [\dot{R}_1]\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + [R_1]\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} \tag{9}$$

$$\{v_{M_6}\} = \begin{bmatrix} \dot{x}_A \\ \dot{y}_A \end{bmatrix} + [\dot{R}_2]\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + [R_2]\begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} \tag{10}$$

where:

$$\begin{aligned} & [\dot{R}_1] = \omega_1\begin{bmatrix} -\sin\varphi_1 & -\cos\varphi_1 \\ \cos\varphi_1 & -\sin\varphi_1 \end{bmatrix} = \omega_1\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}[R_1] = \omega_1[\Omega][R_1], \\ & [\dot{R}_2] = \omega_2\begin{bmatrix} -\sin\varphi_2 & -\cos\varphi_2 \\ \cos\varphi_2 & -\sin\varphi_2 \end{bmatrix} = \omega_2\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}[R_2] = \omega_2[\Omega][R_2]. \end{aligned} \tag{11}$$

By replacing in (9) and (10) the expressions (11) we are obtaining:

$$\{v_{M_6}\} = \omega_1[\Omega][R_1]\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \omega_1[R_1]\begin{bmatrix} \dfrac{\mathrm{d}x_1}{\mathrm{d}\varphi_1} \\ \dfrac{\mathrm{d}y_1}{\mathrm{d}\varphi_1} \end{bmatrix} \tag{12}$$

and

$$\{v_{M_6}\} = \omega_1\begin{bmatrix} \dfrac{\mathrm{d}x_A}{\mathrm{d}\varphi_1} \\ \dfrac{\mathrm{d}y_A}{\mathrm{d}\varphi_1} \end{bmatrix} + \omega_2[\Omega][R_2]\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \omega_2[R_2]\begin{bmatrix} \dfrac{\mathrm{d}x_2}{\mathrm{d}\varphi_1} \\ \dfrac{\mathrm{d}y_2}{\mathrm{d}\varphi_1} \end{bmatrix} \tag{13}$$

By comparing the relations (3) with (12) and by identifying, we obtain:

$$\{v_{M_1}\} = \omega_1 [\Omega][R_1] \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \; \{v_{M_6 M_1}\} = \omega_1 [R_1] \begin{bmatrix} \dfrac{\mathrm{d}\, x_1}{\mathrm{d}\, \varphi_1} \\ \dfrac{\mathrm{d}\, y_1}{\mathrm{d}\, \varphi_1} \end{bmatrix}. \tag{14}$$

By comparing the relations (3) with (13) and by identifying, we obtain:

$$\{v_{M_2}\} = \omega_1 \begin{bmatrix} \dfrac{\mathrm{d}\, x_A}{\mathrm{d}\, \varphi_1} \\ \dfrac{\mathrm{d}\, y_A}{\mathrm{d}\, \varphi_1} \end{bmatrix} + \omega_2 [\Omega][R_2] \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}, \; \{v_{M_6 M_2}\} = \omega_2 [R_2] \begin{bmatrix} \dfrac{\mathrm{d}\, x_2}{\mathrm{d}\, \varphi_1} \\ \dfrac{\mathrm{d}\, y_2}{\mathrm{d}\, \varphi_1} \end{bmatrix} \tag{15}$$

4. Dynamical aspects

For determining the components of the reaction from the contact point between the cam and the follower, we are going to isolate the elements of the mechanism from fig.1.
In fig.3 we are presenting the isolated components. The reduction of forces (M_{red_3}) it is done at the crank of mechanisms 3, of the quadrilateral articulated mechanisms.

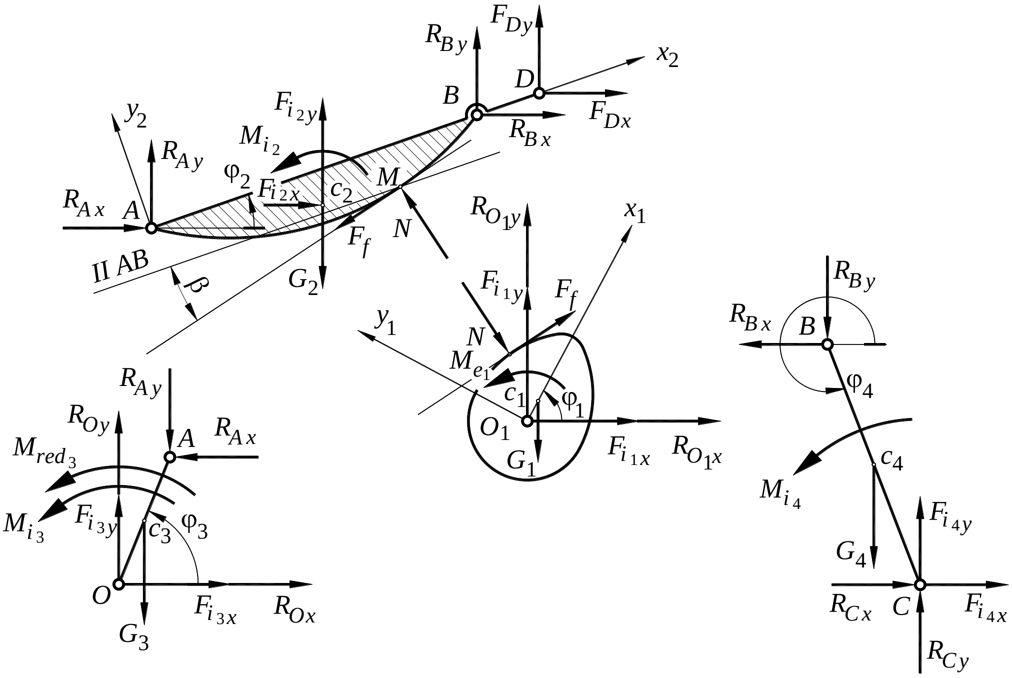

Figure 3. Isolating the elements.

The expression of the moment of reduced inertia M_{red_1} according with [6] is:

$$M_{red_1} = \frac{1}{\omega_3}\left(-G_1 v_{c_{1y}} - G_2 v_{c_{2y}} - G_3 v_{c_{3y}} - G_1 v_{c_{4y}} + F_{D_x} v_{D_x} + F_{D_y} v_{D_y}\right) \tag{16}$$

For obtaining the components of the elastic force given by the spring (fig. 1) the following relation is written:

$$\vec{F}_e = F_e \frac{\overline{DE}}{\left|\overline{DE}\right|} = k(l_{DE} - l_0) \frac{(x_E - x_D)\vec{i} + (y_E - y_D)\vec{j}}{\sqrt{(x_E - x_D)^2 + (y_E - y_D)^2}} \tag{17}$$

from where are obtained the components of the elasical force from point D:

$$F_{D_x} = k(l_{DE} - l_0) \frac{(x_E - x_D)}{\sqrt{(x_E - x_D)^2 + (y_E - y_D)^2}}, \; F_{D_y} = k(l_{DE} - l_0) \frac{(y_E - y_D)}{\sqrt{(x_E - x_D)^2 + (y_E - y_D)^2}}. \tag{18}$$

From the dynamic equilibrium equations, the following equations are obtained:

- for the body 3 $\sum M_O = 0$:

$$R_{A_x}(y_A - y_O) - R_{A_y}(x_A - x_O) - G_3(x_{c_3} - x_O) + M_{red_3} + M_{i_3} = 0, \tag{19}$$

- for the body 4 $\sum M_C = 0$:

$$R_{B_x}(y_B - y_C) + R_{B_y}(x_C - x_B) + G_4(x_C - x_{c_4}) + M_{i_4} = 0, \tag{20}$$

- for the body 2 $\sum F_{x_2} = 0$:

$$\left(R_{A_x} + F_{i_{2x}} + R_{B_x} + F_{D_x}\right)\cos\varphi_2 + \left(R_{A_y} + F_{i_{2y}} + R_{B_y} + F_{D_y}\right)\sin\varphi_2 - F_f\cos\beta - N\sin\beta = 0, \tag{21}$$

- for the body 2 $\sum F_{y_2} = 0$:

$$-\left(R_{A_x} + F_{i_{2x}} + R_{B_x} + F_{D_x}\right)\sin\varphi_2 + \left(R_{A_y} + F_{i_{2y}} + R_{B_y} + F_{D_y} - G_2\right)\cos\varphi_2 - F_f\sin\beta + N\cos\beta = 0, \tag{22}$$

- for the body 2 $\sum M_B = 0$:

$$\begin{aligned} &R_{A_x}(y_B - y_A) - R_{A_y}(x_B - x_A) + M_{i_2} + F_{i_{2x}}(y_B - y_{c_2}) + \left(G_2 - F_{i_{2y}}\right)(x_B - x_{c_2}) + \\ &\qquad + \left(F_f\sin\beta - N\cos\beta\right)(l_{AB} - x_2) - \left(N\sin\beta + F_f\cos\beta\right)y_2 = 0 \end{aligned} \tag{23}$$

In previous formulas, the angle β is given by the friction work with a line parallel with *BC*.

The equations (19) ÷ (23) are making a linear 5 equations system with the unknowns: R_{A_x}, R_{A_y}, F_{B_x}, R_{B_y} and N, that can be written in their matrix form:

$$[A]\{R\} = \{M\} \tag{24}$$

where:

$$[A] = \begin{bmatrix} y_A - y_O & -(x_A - x_O) & 0 & 0 & 0 \\ 0 & 0 & y_B - y_C & x_C - x_B & 0 \\ \cos\varphi_2 & \sin\varphi_2 & \cos\varphi_2 & \sin\varphi_2 & -\mu\cos\beta - \sin\beta \\ -\sin\varphi_2 & \cos\varphi_2 & -\sin\varphi_2 & \cos\varphi_2 & \cos\beta - \mu\sin\beta \\ y_B - y_A & -(x_B - x_A) & 0 & 0 & a_{55} \end{bmatrix}, \tag{25}$$

cu $a_{55} = (\mu\sin\beta - \cos\beta)(l_{AB} - x_2) - (\sin\beta + \mu\cos\beta)$

$$\{R\} = \left(R_{A_x} \quad R_{A_y} \quad R_{B_x} \quad R_{B_y} \quad N\right)^T \tag{26}$$

$$\{M\} = \begin{bmatrix} G_3(x_{c_3} - x_O) - M_{red_3} - M_{i_3} \\ -G_4(x_C - x_{c_4}) - M_{i_4} \\ -\left(F_{i_{2x}} + F_{D_x}\right)\cos\varphi_2 - \left(F_{i_{2y}} + F_{D_y}\right)\sin\varphi_2 \\ \left(F_{i_{2x}} + F_{D_x}\right)\sin\varphi_2 - \left(F_{i_{2y}} + F_{D_y} - G_2\right)\cos\varphi_2 \\ -M_{i_2} - F_{i_{2x}}(y_B - y_{c_2}) - \left(G_2 - F_{i_{2y}}\right)(x_B - x_{c_2}) \end{bmatrix} \tag{27}$$

The solution of system (24) is:

$$\{R\} = [A]^{-1}\{M\} \tag{28}$$

So, there are obtained the values of the normal reaction *N* depending on the position of the motor element.

5. Calculating the mechanical friction work

According to the relation (1), the basic mechanical friction work of *L*, in the general reference system *OXY*, depends on the value of the sliding friction coefficient μ, of value N, of the normal reaction in the contact point and of value v_r of the relative velocity between the cam and the follower.

By definition, the absolute velocity is given by the relation:

$$\vec{v}_a = \vec{v}_t + \vec{v}_r \tag{29}$$

which is written for the couple from the contact *M* between the cam and follower, becomes:

$\vec{v}_{M_2} = \vec{v}_{M_1} + \vec{v}_{M_2 M_1}$ (30)

As mentioned, in the cam-follower contact point *M*, there are two relative velocities: $\vec{v}_{M_1 M_6}$ which highlight the displacement the contact on the cam, and $\vec{v}_{M_2 M_6}$ which highlights the displacement of the contact point on the follower. Both relative velocities have the direction of the tangent vector in *M*, for both curves.

The relation (3) can be written under the form:

$\vec{v}_{M_2} = \vec{v}_{M_1} + \vec{v}_{M_6 M_1} - \vec{v}_{M_6 M_2}$. (31)

By comparing the relations (30) and (31) we can write that:

$\vec{v}_r = \vec{v}_{M_2 M_1} = \vec{v}_{M_6 M_1} - \vec{v}_{M_6 M_2}$. (32)

As the relative velocities $\vec{v}_{M_1 M_6}$ and $\vec{v}_{M_2 M_6}$ are collinear, the module of the relative velocity is:

$v_r = v_{M_6 M_1} - v_{M_6 M_2}$, (33)

which also results from the velocities polygon from fig. 2.

Considering that the angular velocity ω_1 is being constant, by integrating we are obtaining the expression of the mechanical friction work:

$$L = \frac{\mu}{\omega} \int_0^{2\pi} |N| \left| v_{M_6 M_1} - v_{M_6 M_2} \right| \mathrm{d}\varphi . \tag{34}$$

If the kinematic analysis is done with a constant angular step of $1°$, the integral (34) is numerically calculated with the relation:

$$L = \frac{\mu}{\omega} \frac{\pi}{180} \sum_{i=1}^{360} |N|_i \left| v_{M_6 M_1} - v_{M_6 M_2} \right|_i$$

6. Conculsions

A very important criterion in assessing the quality of a cam mechanism is its efficiency. The efficiency is dependent of the value of the mechanical friction work from the cam-follower couple. Its size is influenced by: the value of sliding friction coefficient, the value of the normal reaction and by value of the relative displacement from the contact point between the cam and the follower.

Highlighting the relative movement of the contact point on the cam and follower it is done by introducing a zero-dimension ring between the two elements. In order to make the future numerical application easier to solve, the relative velocities from the cam-follower contact point are presented under their matrix form.

To obtain the normal reaction value, we isolate the elements of the articulated quadrilateral mechanism and of the cam. 5 equations with 5 unknowns are retained from the dynamic equilibrium equations, from which will result the numerical values of the reactions from two kinematic couplings of the articulated quadrilateral mechanism, and the value of the normal reaction from the cam-follower coupling. The values necessary for calculating the mechanical friction work are thus obtained.

A numerical application will be done in a future paper, that will explain the calculus presented in this paper.

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